A SIMPLIFIED APPROACH TO PREDICT SURFACE RUNOFF AND WATER LOSS HYDROGRAPHS

By

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Abstract. A generalized mathematical model with three parameters, P, α and β, has been developed to predict surface runoff and water loss hydrographs from gaged and ungaged watersheds. The parameter P, found to be the most important by sensitivity analysis, was correlated to rainfall and watershed characteristics and time of the year in a Fourier series expression with four terms. The parameter α, second in order of importance, was correlated to rainfall hyetograph characteristics and watershed area. The parameter β is expressed as a function of α in a fifth order polynomial based upon Legendre approximation. The surface runoff hydrographs obtained by using the model approximated very well the observed surface runoff hydrographs consisting of one, two, and more than two peaks.

KEY WORDS: mathematical model; watershed; surface runoff; water loss.

INTRODUCTION

An ability to predict the reactions of a watershed to the complicated input, rainfall, is one of the key factors in watershed management. Availability of high speed computers and advancements in science have encouraged the development of complex models which, in general, involve a large number of parameters. Difficulties in estimating large numbers of parameters arise quite frequently, due not only to unavailability of long term hydrologic records, but also to the complexity of numerous soil, climatic and human factors affecting them spatially and temporally. Therefore, approximations that eliminate some of the elements but which are based upon a minimum number of factors that incorporate the excluded ones are desired.

A considerable amount of work has been done to predict watershed reactions from the application of real or hypothetical storms. Some of them, for example, are as reported by Crawford and Linsley (1966), Huggins and Monke (1966), Holtan and Lopez (1971), Dooge (1959), Nash (1958), Overton (1967), Laurenson (1964), Dawdy et al.(1970) and Prasad (1967). This paper presents an overall model to predict the reactions of gaged or ungaged watersheds in terms of surface runoff and water loss hydrographs. The development of the model and associated simplifying assumptions are discussed hereafter.

MODEL DEVELOPMENT

A good qualitative understanding of the runoff process is necessary to delineate or lump the parts of the process for which explicit mathematical relationships are to be derived. For this purpose, a conceptual model used to show the disposition of rainfall as it reaches the earth surface is presented in Figure 1. When rainfall begins, a portion of the water is intercepted by plant surfaces and never reaches the ground. Water that reaches the ground surface may infiltrate immediately or it may flow into such temporary storages as soil fissures and surface depressions to evaporate or infiltrate later. The water moves off the surface as overland flow after the composite demands for interception, infiltration, depression storage, and surface detention become less than the rate of supply. The portion of surface detention which does not infiltrate and evaporate becomes overland flow after the rainfall ceases. The water that infiltrates either percolates through the soil profile to the groundwater, is stored as soil moisture in the zone of aeration, is lost by evapotranspiration, or moves as interflow to later reappear on the surface at other points within or outside the watershed boundaries. (Fig. 1

The rainfall reaching the watershed surface can, therefore, be grouped into two major categories. One category includes the portion of rainfall that is not observed at the watershed outlet and is here referred to as water loss from watersheds. Another category includes the difference between rainfall and water loss and is here referred to as rainfall excess. With this as a background it is hypothesized that

> "there exists a parameter, P, which transforms the rainfall over a watershed surface into rainfall excess and water loss. This parameter may be a function of rainfall and watershed characteristics, and time of the year which may act as an index to the watershed conditions."

The transformation of rainfall over a watershed into rainfall excess and water loss is undoubtedly very complex. However, using the work of

Crawford and Linsley (1966) and Dawdy, et al. (1970) as insight, and recognizing watershed heterogeneity, the rate of water loss from a given supply rate of rainfall over a watershed is assumed to be linear from 0 to P and takes the form (for rainfall $\leq$ P) that is presented in Figure 2. Fig.2

L/T in Figure 2 is a dimension of rate where L is distance and T is time. Mathematical relationships for determining water loss and rainfall excess can easily be derived from Figure 2 in terms of supply rate of rainfall, R, and parameter, P. These relationships are as follows:

For $R \leq P$,

$$WL = R - \frac{R^2}{2P} \quad (1)$$

$$RE = \frac{R^2}{2P} \quad (2)$$

For $R \geq P$,

$$WL = \frac{P}{2} \quad (3)$$

$$RE = R - \frac{P}{2}$$

where WL is water loss and RE is rainfall excess.

Physically, the parameter P may be viewed with a restriction as an indicator of the rate of water loss occurring from a rainfall event on a watershed. The restriction is that it must distribute rainfall in any selected time interval into rainfall excess and water loss such that at the end of the rainfall storm the total rainfall excess is matched with total observed surface runoff within a specified tolerance. Also, the parameter P sets a maximum limit as one-half of its value for the rate at which water loss can occur in any time interval of the rainfall storm under consideration.

Having determined the rainfall excess that would result from an input rainfall, the remaining problem is to distribute the rainfall excess over time; that is, watershed storage routing. For this purpose it is assumed that storage, S, in the watershed is a function of discharge, Q, alone and the two are related as

$$S = K(Q) \cdot Q \qquad (5)$$

where K is a time parameter and from now onwards, K will be used to represent K(Q).

Using Laurenson's (1964) analogy of K to the lag, work of Crawford and Linsley (1966), and the assumption that K is a function of discharge, it may be inferred that the relationship between K and Q is of the form

$$K = \alpha Q^{\beta} \qquad (6)$$

where α and β are shape factors.

Substituting rainfall excess, RE, for average inflow in the finite difference form of the continuity equation and rearranging, gives

$$(RE)\ \Delta t - (Q_2 + Q_1)\ \frac{\Delta t}{2} = \Delta S = S_2 - S_1 \qquad (7)$$

where subscripts $_1$ and $_2$ refer respectively to the beginning and end of the routing period, Δ means a change, and t is time.

Substituting equation 6 into equation 7 and solving for Q_2 yield

$$Q_2 = CO(RE) + C2(Q_1) \qquad (8)$$

where $CO = \dfrac{2\Delta t}{2K_2+\Delta t}$ and $C2 = \dfrac{2K_1-\Delta t}{2K_2+\Delta t}$

Equation 8 is the storage routing function. However, an alternate storage routing equation can be obtained in a differential equation form by combining the storage function with the continuity equation. The continuity equation in the differential equation form can be written as

$$RE(t) - Q(t) = \frac{ds(t)}{dt} \tag{9}$$

where $\frac{d}{dt}$ is the change with respect to time.

Combining equations 5 and 6 with equation 9 and rearranging, gives

$$RE(t) = Q(t) + \alpha(Q[t])^{\beta} \frac{dQ(t)}{dt} \tag{10}$$

Equation 10 is a first order nonlinear differential equation describing the response of a time invariant, lumped, first order nonlinear system. Equation 10, hereafter referred to as an alternate routing function, can be used to describe the relationship between the input RE and output Q from a basin at time, t. Equation 10 may be written in terms of a differential operator, D, as

$$Q(t) = \frac{RE(t)}{\alpha(Q[t])^{\beta}D + 1} \tag{11}$$

in which $\frac{1}{\alpha(Q[t])^{\beta}D + 1}$ is the nonlinear operator which transforms the input RE into the output Q.

A solution to equation 10 by a numerical integration technique would result in

$$Q_{i+1} = Q_i + (\dot{Q}_i + \dot{Q}_{i+1}) \frac{\Delta t}{2} \tag{12}$$

where $\dot{Q} = \frac{dQ(t)}{dt}$ and i and i+1 are the beginning and end of the time step. A detailed derivation of equation 12 is given by Sinha (1970).

Dropping t for convenience of writing, equation 10 can be rewritten as

$$\dot{Q} = \frac{RE - Q}{\alpha(Q)^{\beta}} \tag{13}$$

If the rainfall excess hyetograph and surface runoff hydrograph are known, then two unknowns, $\dot{Q}_{i+1}$ and Q_{i+1}, in equation 12 can be found for a given value of α and β by using equations 12 and 13 in an iterative procedure.

The purpose of developing an alternate routing equation is to illustrate that, starting with a basic continuity equation and storage function, a worker may develop a simple algebraic equation like equation 8 as well as a somewhat complicated nonlinear differential equation like equation 10. Regardless of the form of the equation, it is necessary that the mathematical model developed for the purpose be tested.

MODEL TESTING

Thirty surface runoff producing rainfalls on fourteen small watersheds varying in area between 0.0025 and 7.1563 square miles were selected from the United States Department of Agriculture (1956-1959, 1960-1961). Twelve of the fourteen watersheds were located at Cochocton, Ohio; one was located at Watkinsville, Georgia; and one was located at LaCrosse, Wisconsin. Rainfall and runoff values were tabulated at two minute intervals. The base flow involved in any of the selected hydrographs was negligible. The three parameters, P, α and β were estimated in the following manner.

<u>Estimation of P.</u> The most important parameter in the model is P because it not only defines water loss and thus controls the total volume of surface runoff that is expected to occur from a particular storm event on a watershed, but also, it produces an inflow hydrograph of rainfall

excess for storage routing. The value of P which would produce the total volume of surface runoff that can be observed from a given pattern of rainfall has been proven analytically by Sinha (1970) to be unique. Thus, a simple iteration technique is used to estimate P for the gaged watershed where rainfall distribution and total volume of surface runoff, ROTO, resulting from this rainfall storm are known. The iteration procedure, which converges fast, involves initializing P as 1 or any other value and then modifying the current value of P by multiplying it with a ratio of ROTC to ROTO until the absolute difference of ROTO and ROTC falls within a specified tolerance. ROTC is the total volume of rainfall excess computed by using equations 2 and 4.

The value of P estimated for each of the thirty storms by the iterative procedure is presented in Table 1. Then, efforts were made to determine P for ungaged watersheds from such input information as rainfall and watershed characteristics and time of the year. Table 1

The rainfall and runoff data used in this study were for seven different months of the year and most of them were for the month of June. Hoping that there may be some sort of an annual cycle made perhaps of frequencies that are fractions of a year, week number was used to represent the time of year in the Fourier series expression which will cycle in a (52-week) year. Use of week number in an analysis of this nature has been made by Betson et al. (1969). It was expected in this study that some combination of three to four terms, which can be justified by the data, in Fourier series would do the job. And it so happened in this study that n = 1, 2, 3, 4 did the job. Physically n = 1, 2, 3, 4 represents annual, half-yearly, one-third period of the year, and quarterly terms, respectively. Thus, an equation to estimate the value of P as a function of time of the year, and rainfall and watershed characteristics, was written as

$$P_j = \sum_{n=1}^{4} a_n \, \mathrm{Sin} \left(\frac{2\pi n[Y-\ell_n]}{52} \right) + bi\, x_i + e_j \qquad (14)$$

where P = value of parameter P obtained by iterative procedure,

j = 1, 2, ..., 29, 30, = serial number for storms,

$\sum$ = summation sign,

n = 1, 2, 3, 4 = number of terms to be used in the series,

a = constant,

Sin = sine,

π = 3.1416 or 180 degrees,

Y = week number, 1, 2, ..., 51, 52,

ℓ = lag,

52 = total number of weeks in a year on which the series would cycle,

b = constant,

i = 1, 2, ... ,

x = variable representing rainfall and watershed characteristics, and

e = error of fit, or random element associated with each value of P.

The constants in equation 14 can be determined by multiple regression and correlation technique. However, before this technique is used the series expression has to be rearranged as follows:

$$\sum_{n=1}^{4} a_n \, \mathrm{Sin} \left(\frac{2\pi n[Y-\ell_n]}{52} \right) = \sum_{n=1}^{4} (r_{2n-1} \, \mathrm{Sin} W_n - r_{2n} \, \mathrm{Cos} W_n)$$

where $r_{2n-1} = a_n \, \text{Cos}M_n$, $r_{2n} = a_n \text{Sin}M_n$

$$M_n = \frac{2\pi n \ell_n}{52}, \quad W_n = \frac{2\pi n Y}{52}$$

Thus, use of multiple regression and correlation techniques would give the constants $r_1, r_2, \ldots, r_7, r_8$ associated with the series terms in equation 14. The values of constants a and ℓ can be determined from the values of r as:

$$\frac{r_{2n}}{r_{2n-1}} = \frac{a_n \text{Sin}M_n}{a_n \text{Cos}M_n}, \quad M_n = \text{Tan}^{-1} \left(\frac{r_{2n}}{r_{2n-1}} \right) \text{ for } n = 1, 2, 3, \text{ and } 4.$$

After knowing the values of M, the values of ℓ and a can easily be determined as:

$$\ell_n = \frac{52 \times M_n}{2\pi n} \text{ and } a_n = \frac{r_{2n}}{\text{Sin}M_n} = \frac{r_{2n-1}}{\text{Cos}M_n} \quad \text{for } n = 1, 2, 3, \text{ and } 4.$$

The coefficients of equation 14 were then determined by using multiple regression and correlation analysis. The statistical results are presented in Table 2 and the resulting equation is as follows: Table 2

$$\begin{aligned} \hat{P} = {} & 0.5758 \, \text{Sin}W_1 - 0.0863\text{Cos}W_1 + 0.3014\text{Sin}W_2 - 0.3889\text{Cos}W_2 \\ & + 0.2065\text{Sin}W_3 - 0.6937\text{Cos}W_3 - 0.3678\text{Sin}W_4 - 0.5362\text{Cos}W_4 \\ & - 0.00001 \, \text{DIST} - 0.0311 \, (\text{RT})^2 \end{aligned} \qquad (15)$$

where $\hat{P}$ = predicted valued of the parameter P,

DIST = horizontal distance between most distant point on the watershed and outlet of the watershed (feet), and

RT = total rainfall (inch).

Equation 15 seems to indicate that the value of parameter P accociated with a rainfall storm on a given watershed in any week of the year would decrease approximately by three-hundredths of the square of total rainfall occurring from that storm. The values of t statistic in Table 2 seem to indicate that the Fourier series terms (cyclic terms) are prominent in explaining variations in P values and dominant among the cyclic terms are the quarterly terms. The dominance of quarterly terms would seem to indicate that there could be four discernible seasons under the watersheds studied, especially in Coshocton, Ohio; and these seasons may have a large effect on the values of parameter P. The values of P predicted by equation 15 for each of the selected thirty storms are presented in Table 1.

Estimation of α and β. The parameters α and β affect the shape of the hydrograph and evidences (1964, 1964-65, 1965-67) indicate that their simultaneous determination is somewhat difficult. Therefore, the values of α and β for each of the selected storms were determined by trial. The trial procedure involved varying one parameter while holding the other two constant. The final estimates of α and β as obtained by trial were those which produced an acceptable fit with the observed hydrograph. The fit was considered acceptable when the model matched the largest value of the observed surface runoff hydrograph within a specified tolerance, routed total volume of surface runoff over a fixed length of time base without any gain or less, and produced the surface runoff values at the end of each time interval fairly close to that of observed values.

The values of α and β obtained by trial for each of the thirty selected storms are presented in Table 1. The final trial values of α were correlated with the rainfall excess characteristics, watershed area, and the values of P. The statistical results thus obtained are presented in Table 3 and the resulting equation is as follows: Table 3

$$\hat{\alpha} = -38.7406 + 20.2252\,(PKRE) + 28.3973\,(RET) + 100.5709(P) + 2935.4238(AREA) - 16.1999\,(RET)^2 + 72.5669(AVRE)^2 + 0.7710\,(AREA)^2 - 6.9585(PKRE \times RET) - 18.9834(PKRE \times AVRE) + 23.4920(PKRE \times P) - 1074.7770(PKRE \times AREA) + 5077.5530(RET \times AREA) - 251.3478(AVRE \times P) - 5277.4894(AVRE \times AREA) - 5988.4912(P \times AREA). \quad (16)$$

where $\hat{\alpha}$ = predicted value of α,

PKRE = peak rate of rainfall excess (in/hr),

RET = total volume of rainfall excess as computed by the value of P for that storm (in),

AVRE = average rainfall excess rate obtained by dividing RET with DRAIN (in/hr). DRAIN is the duration of rainfall (hour) and includes the number of intervals only in which it rained, and

AREA = watershed area (square miles).

Using equation 16, the values of α for each of the thirty storms were regenerated and these values are also presented in Table 1.

The values of t statistic presented in Table 3 seem to indicate that dominant among the selected variables affecting the values of α are the linear term of watershed area, and linear x linear interaction terms of watershed area with PKRE, RET, AVRE and P.

While finding the values of α and β by trial it was observed that the value of β was influenced by the chosen value of α. Therefore, a curve fitting was done between values of β and α obtained by trial. A fifth order polynomial based upon Legendre approximation was found satisfactory and the resulting equation is

$$\beta = -0.3449 + 0.0354(\alpha) - 0.00149(\alpha)^2 + 0.000029(\alpha)^3 - 0.00000026(\alpha)^4 + 0.0000000009(\alpha)^5 \qquad (17)$$

Using equation 17, the values of β for each of the thirty storms were regenerated and these values are presented in Table 1.

Simulation. The value of P as computed by equation 15 was negative for the storm of 6-28-57 on watershed number 115; it was therefore eliminated from simulation. A negative value of P means that R cannot be $\leq$ P and for R $\geq$ P, RE = R - $\frac{P}{2}$ is positive, but RE will be greater than R. Such happenings are physically impossible. Out of twenty-nine storms simulated, nine storms have one peak, nine storms have two peaks, and eleven storms have more than two peaks. Simulation was done by using both the routing equations 8 and 10. The results of simulation for all of the twenty-nine storms in terms of total volume of surface runoff, peak rate (highest discharge rate in case of hydrographs with more than one peak), and time to the peak rate are summarized in Table 4. The entire hydrograph for only three of the storms is presented in Figures 3 through 5. No water loss hydrographs are presented here but they are those obtained by using equations 1 and 3.

Tab
4
Fig.
4

The ability of equation 10 to simulate total volume of observed surface runoff, peak rate, time to peak and time distribution of surface runoff; that is, the entire surface runoff hydrograph, appeared superior as compared to that of equation 8. This may be due to the use of observed surface runoff values in equation 10. A distinct disadvantage with equation 10 is, therefore, that the observed surface runoff hydrograph resulting from every storm must be known. The advantage with equation 8 is that if reliable equations based upon rainfall and watershed characteristics and time of year are developed to produce the values of P, α, and β, then hydrographs for similar ungaged watersheds may be developed. The possibility of developing such reliable equations is very well exemplified with equations 15, 16 and 17. Apparent from the results is that the model, in general, is well suited to watersheds of areas up to 0.2 square mile, with flat to moderate slopes. A sensitivity analysis indicated that the parameter P is most important and next in order is the parameter α, while β seems to be the least important.

SUMMARY AND CONCLUSIONS

A generalized mathematical model with three parameters has been developed to predict surface runoff and water loss hydrographs from gaged and ungaged watersheds. The values of parameter P, which determine the total volume of surface runoff and water loss resulting from given rainfall storms, were estimated by an iterative procedure and then were correlated to rainfall and watershed characteristics and week of the year so that the model could be applied to ungaged watersheds. The 't' statistic indicated that cyclic terms were prominent in explaining the variations in the values of P, and dominant among the cyclic terms was the quarterly term.

The shape parameters α and β were determined by trial. The values of α so determined were correlated to the characteristics of rainfall excess hyetograph and the watershed area and the values of P. The 't' statistic seemed to indicate that dominant among the selected factors affecting the values of α were the linear term of watershed area, and linear x linear interaction terms of watershed area with PKRE, RET, AVRE and P. A fifth order polynomial based upon Legendre approximation was developed to determine the values of β from the values of α.

Very high R^2 values of 0.92 and 0.99 associated with equations 15 and 16, respectively, indicate that equations to predict the values of parameters P and α can be developed from such input information as rainfall, watershed characteristics and time of the year. The surface runoff hydrographs obtained by using the model approximated very well the observed surface runoff hydrographs consisting of one peak, two peaks, and more than two peaks. Apparent from the results is that the model is well suited to watersheds with areas up to 0.2 square mile with flat to moderate slopes. Water loss hydrograph resulting from a rainfall storm will be that obtained by using equations 1 and 3 and an appropriate value of parameter P. A sensitivity analysis indicated that P was the most important parameter, and next in order of importance was the parameter α while the parameter β seemed to be the least important.

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LIST OF FIGURES

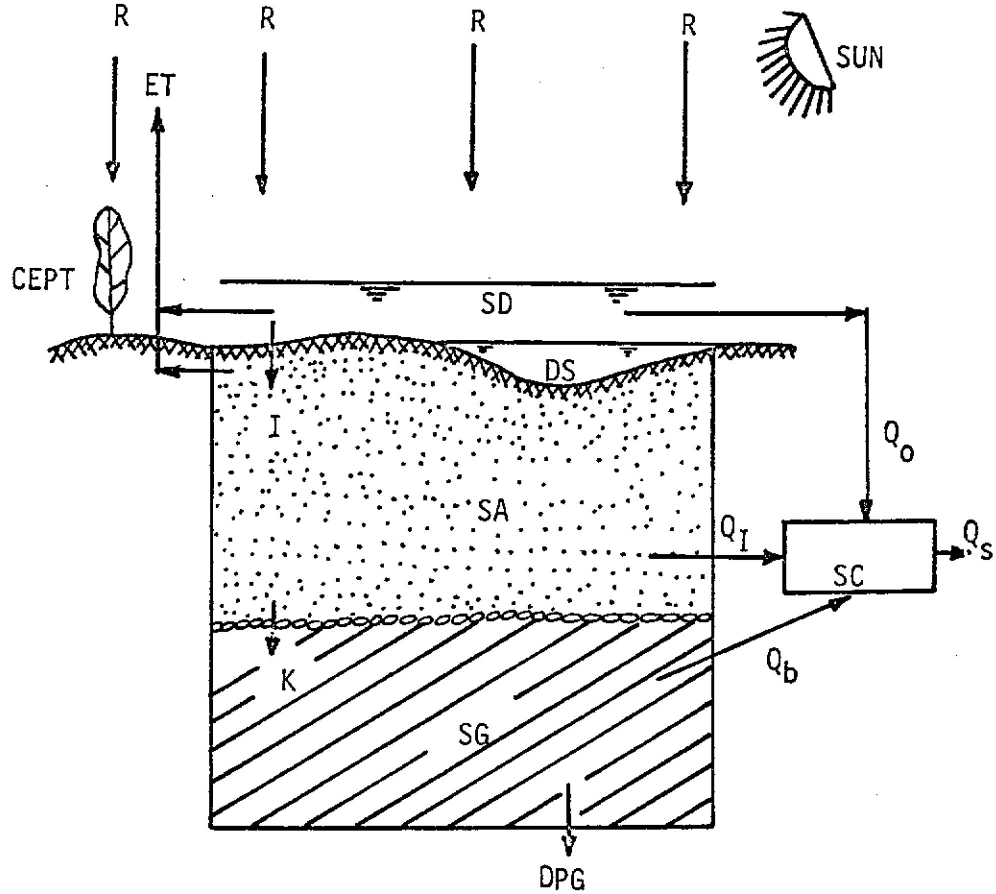
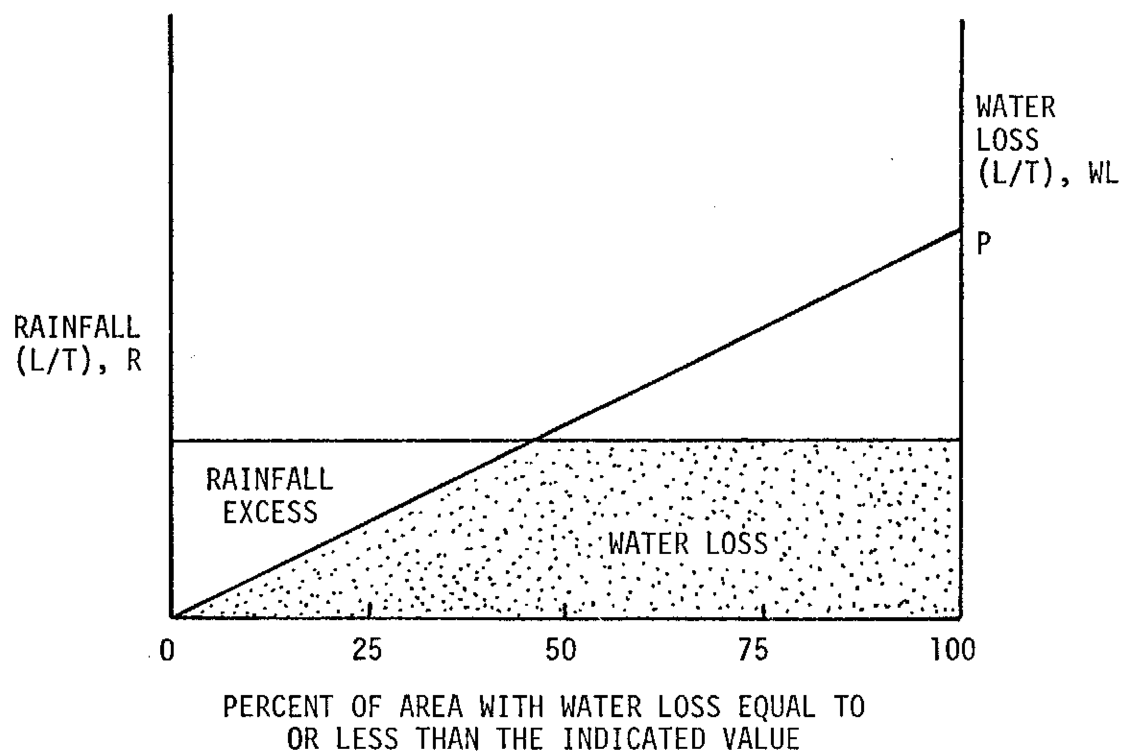
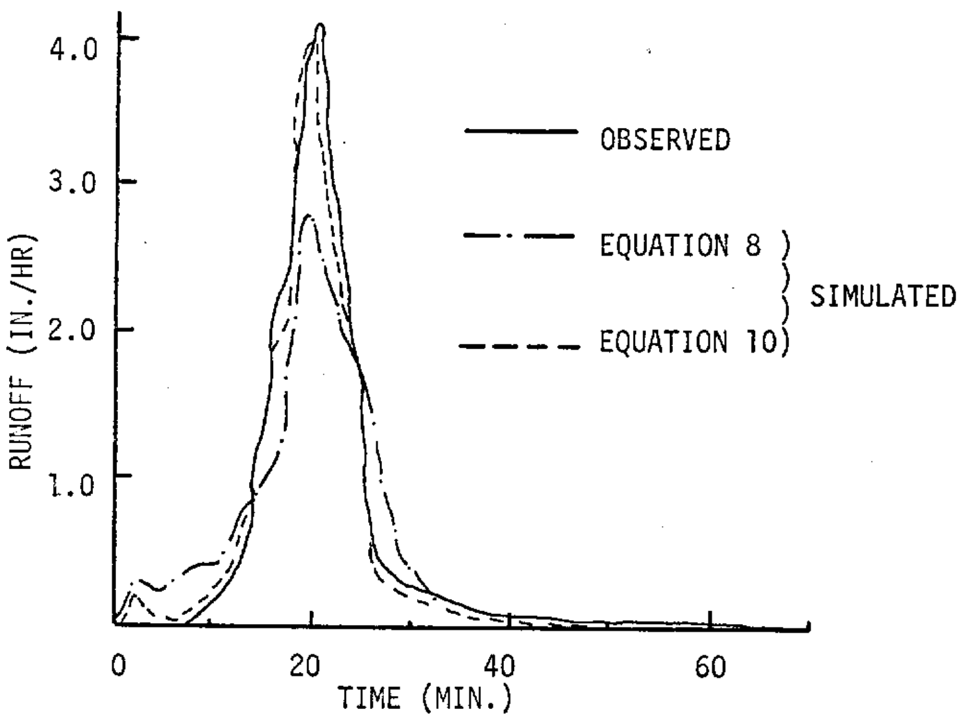


Figure 1. Disposition of rainfall on the earth

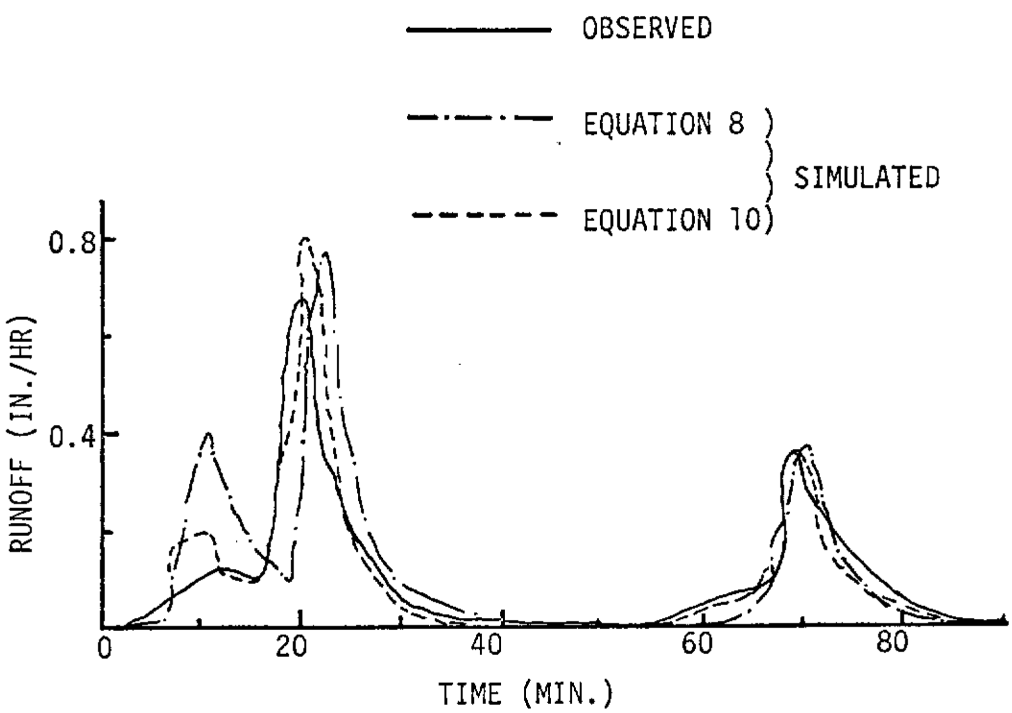
R = Rainfall	SG = Groundwater storage
ET = Evapotranspiration	DS = Depression storage
I = Infiltration	xxxxx = Earth surface
K = Percolation	SC = Channel storage
Q_o = Overland flow	CEPT = Interception
Q_b = Base flow	SD = Surface detention
Q_I = Interflow	∞∞ = Groundwater table
Q_S = Streamflow	DPG = Deep percolated groundwater

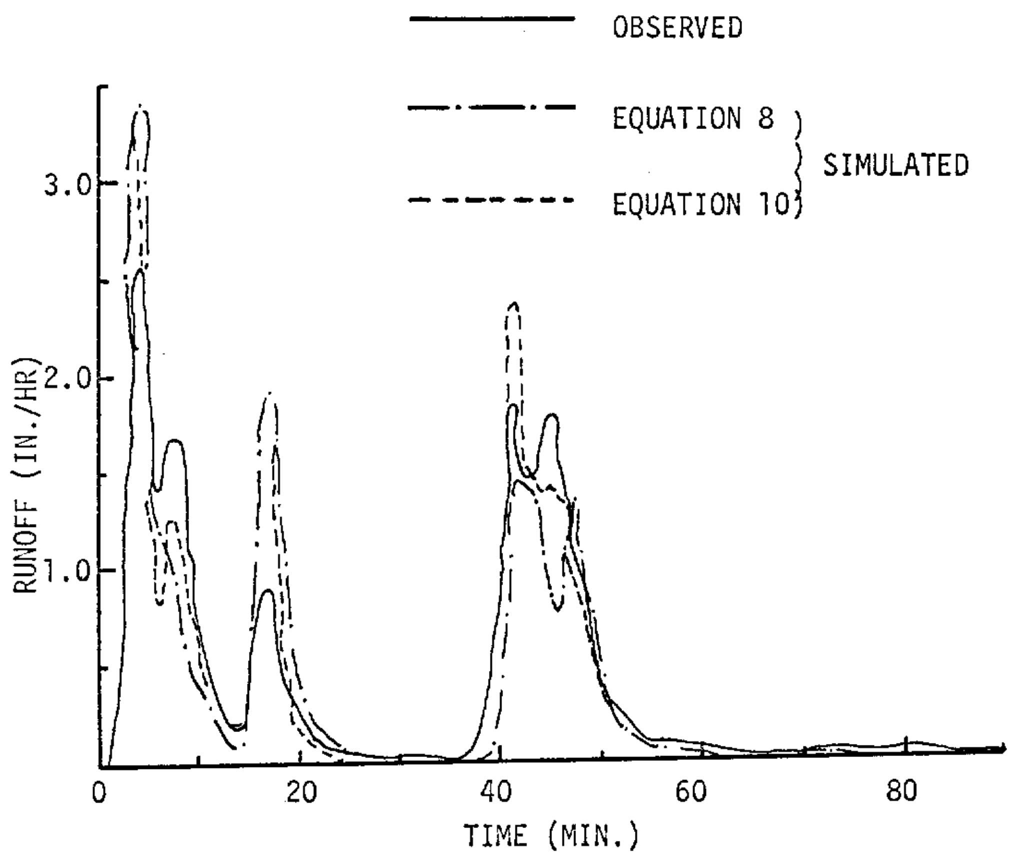
SA = Storage in the zone of aeration


WATER
LOSS
(L/T), WL
P
RAINFALL
(L/T), R
RAINFALL
EXCESS
WATER LOSS
0
25
50
75
100
PERCENT OF AREA WITH WATER LOSS EQUAL TO
OR LESS THAN THE INDICATED VALUE



4.0
3.0
2.0
1.0
RUNOFF (IN./HR)
0
20
40
60
TIME (MIN.)
OBSERVED
EQUATION 8
EQUATION 10
SIMULATED


OBSERVED
EQUATION 8
SIMULATED
EQUATION 10
0.8
0.4
RUNOFF (IN./HR)
0
20
40
60
80
TIME (MIN.)



OBSERVED
EQUATION 8
EQUATION 10
SIMULATED
RUNOFF (IN./HR)
3.0
2.0
1.0
0
20
40
60
80
TIME (MIN.)

Table 1. Information concerning values of parameters P, α and β.

Serial Number of Storm	Watershed Number (1)	Storm Date (2)	Value of P by Iterative Procedure (3)	Value of P by Equation 15 (4)	Value of α by Trial (5)	Value of α by Equation 16 (6)	Value of β by Trial (7)	Value of β by Equation 17 (8)
Coshocton, Ohio								
1	115	6-12-57	0.15723	0.18520	6.15	6.3289	-0.28	0.0087
2	115	6-28-57	0.02871	-0.03622	4.42	4.1737	-0.283	-0.0809
3	115	9-23-45	0.04586	0.10514	6.9	13.2336	-0.09	-0.0127
4	118	6-12-57	0.025921	0.16936	8.8	9.8861	-0.07	-0.0335
5	118	9-23-45	0.04447	0.05733	20.23	16.4341	-0.09	-0.0241
6	129	6-12-57	0.21061	0.17069	4.77	3.9815	-0.06	-0.1619
7	129	6-28-57	0.03238	0.03850	12.96	11.6220	-0.008	-0.0413
8	129	9-23-45	0.16662	0.11449	6.834	4.8725	-0.05	-0.036
9	130	6-12-57	0.14593	0.14245	3.89	1.7110	-0.2	-0.0377
10	130	6-28-57	0.02005	0.03487	5.89	5.0206	-0.157	-0.0324
11	130	9-23-45	0.10879	0.10288	17.192	21.4586	-0.15	-0.0833
12	135	6-12-57	0.21808	0.17643	4.5	5.5425	-0.32	-0.1641
13	135	6-28-57	0.04178	0.03839	12.7	10.3067	-0.05	-0.2071
14	135	9-23-45	0.12947	0.11538	6.984	6.7001	-0.1	-0.0865
15	115	6-28-40	0.04040	0.04610	7.1254	3.5461	-0.205	-0.1309
16	118	6-28-40	0.02933	0.07101	6.31	8.1612	-0.08	-0.1742
17	129	8-21-60	0.21005	0.29427	6.954	5.1371	-0.035	-0.1797
18	130	4-25-61	0.04519	0.04197	9.64	8.6381	-0.12	-0.0404
19	135	8-21-60	0.37611	0.29189	6.035	10.0803	-0.08	-0.1629
20	135	4-25-61	0.05890	0.06212	7.5	9.5955	-0.093	-0.1521
21	177	6-12-57	0.13655	0.10817	12.37	14.9395	-0.07	-0.1614
22	10	6-12-57	0.25352	0.24974	50.0	45.0182	-0.02	-0.1187
23	5	6-12-57	0.31334	0.26296	26.65	23.0755	-0.05	-0.0518
24	92	6-12-57	0.30725	0.24775	46.9	48.0944	-0.04	-0.1827
25	94	6-12-57	0.17251	0.17087	57.4	59.8456	-0.03	-0.2255
26	95	6-12-57	0.15093	0.21115	90.55	88.9040	-0.01	-0.2282
27	97	6-12-57	0.16706	0.16416	104.385	4.6328	-0.007	-0.1588
Watkinsville, Georgia								
28	W-1	5-15-42	0.06120	0.06120	19.9	22.6592	-0.04	-0.1774
29	W-1	1-29-60	0.13200	0.13200	42.0	42.3477	-0.045	-0.2153
LaCrosse, Wisconsin								
30	CWA	7-19-52	0.08079	0.08079	4.0	3.9923	-0.1924	-0.2134

Table 2. Statistical results from multiple regression analysis of parameter P.

Coefficient of multiple correlation, $R^2 = 0.92$
Standard error of dependent variable, $\sigma = 0.0558$
Total degrees of freedom = 30
Regression degrees of freedom = 10
Error degrees of freedom = 20
Variance ratio of regression to error, F value = 23.04

Variable	Coefficient		Standard error of coefficient	Student's t
$\sin \frac{2\pi y}{52}$	0.5758	$a_1 = 0.5822$	0.1076	5.3528***
		$\ell_1 = 51$		
$\cos \frac{2\pi y}{52}$	-0.0863		0.0933	-0.9242
$\sin \frac{2\pi(2)y}{52}$	0.3014	$a_2 = 0.4919$	0.0553	3.4524***
$\cos \frac{2\pi(2)y}{52}$	-0.3889	$\ell_2 = 22$	0.1557	-2.4983**
$\sin \frac{2\pi(3)y}{52}$	0.2065	$a_3 = 0.7235$	0.0520	3.9683***
$\cos \frac{2\pi(3)y}{52}$	-0.6937	$\ell_3 = 14$	0.1958	-3.5426***
$\sin \frac{2\pi(4)y}{52}$	-0.3678	$a_4 = 0.6502$	0.0641	-5.7336***
$\cos \frac{2\pi(4)y}{52}$	-0.5362	$\ell_4 = 9$	0.1505	-3.5628***
DIST	-0.000010		0.000005	-1.9767*
$(RT)^2$	-0.0311		0.0096	-3.2341***

* Hereafter single asterisk means significant at a probability level less than or equal to 0.10.
**Hereafter double asterisk means significant at a probability level less than or equal to 0.05.
***Hereafter triple asterisk means significant at a probability level less than or equal to 0.01.

Table 3. Statistical results from multiple regression analysis of parameter α.

Coefficient of multiple correlation, $R^2 = 0.99$
Standard error of dependent variable, $\sigma = 3.7212$
Regression constant or intercept = -38.7406
Total degrees of freedom = 29
Regression degrees of freedom = 15
Error degrees of freedom = 14
Variance ratio of regression to error, F value = 90.77

Variable	Coefficient	Standard error of coefficient	Student's t
PKRE	20.2252	3.5353	5.7210***
RET	28.3973	12.3404	2.3012**
P	100.5709	26.5896	3.7823***
AREA	2935.4238	302.2130	9.7131***
$(RET)^2$	-16.1999	6.0384	-2.6828**
$(AVRE)^2$	72.5669	14.5513	4.9870***
$(AREA)^2$	0.7710	0.3941	1.9565*
PKRE x RET	-6.9585	4.0983	-1.6979
PKRE x AVRE	-18.9834	5.9065	-3.2140***
PKRE x P	23.4920	9.8845	2.3766**
PKRE x AREA	-1074.7770	109.5971	-9.8066***
RET x AREA	5077.5530	586.0933	8.6634***
AVRE x P	-251.3478	66.3353	-3.7890
AVRE x AREA	-5277.4894	738.3198	-7.1480***
P x AREA	-5988.4912	621.4242	-9.6367

Table 4. Total volume of surface runoff, peak rate, and time to peak rate

Serial Number of Storm	Total volume of surface runoff (in.)				Peak rate (in./hr.)			Time to peak rate from beginning of surface runoff (min.)		
		Predicted	Routed			Simulated			Simulation	
	Observed (1)	Equation 15 (2)	Equation 8 (3)	Equation 10 (4)	Observed (5)	Equation 8 (6)	Equation 10 (7)	Observed (8)	Equation 8 (9)	Equation 10 (10)
1	1.21	1.05	1.05	1.16	4.12	2.84	3.98	40	38	38
3	0.83	0.42	0.41	0.77	1.55	0.62	1.59	40	40	40
4	1.03	1.48	1.47	1.11	3.11	3.88	3.17	36	44	36
5	1.10	0.93	0.93	1.08	1.36	1.32	1.35	58	52	58
6	1.02	1.25	1.25	1.12	2.36	2.99	2.67	38	38	38
7	0.80	0.70	0.70	0.78	1.10	1.09	1.19	110	110	110
8	0.24	0.35	0.35	0.28	0.527	0.91	0.76	40	44	40
9	1.48	1.51	1.51	1.50	4.06	4.19	4.08	36	34	34
10	1.02	0.72	0.72	0.92	1.43	1.22	1.38	140	140	140
11	0.47	0.50	0.49	0.47	0.852	0.75	0.78	44	40	44
12	0.96	1.18	1.18	1.03	2.38	2.81	2.49	40	38	38
13	0.66	0.70	0.70	0.67	1.01	1.23	1.12	110	110	110
14	0.31	0.35	0.35	0.32	0.678	0.78	0.817	40	44	40
15	1.16	1.11	1.11	1.15	2.57	3.40	3.29	8	8	6
16	1.04	0.71	0.71	0.98	1.93	1.30	1.82	8	10	8
17	0.20	0.14	0.14	0.18	0.556	0.44	0.555	24	24	22
18	0.81	0.84	0.84	0.82	1.23	1.30	1.09	86	96	88
19	0.11	0.14	0.13	0.12	0.32	0.29	0.37	24	24	22
20	0.67	0.64	0.63	0.66	1.32	1.13	1.07	64	78	64
21	1.51	1.74	1.74	1.50	3.14	3.45	2.75	50	36	50
22	0.43	0.44	0.43	0.43	0.329	0.337	0.317	96	20	96
23	0.35	0.42	0.42	0.35	0.432	0.58	0.40	64	18	64
24	0.36	0.44	0.44	0.36	0.28	0.30	0.27	34	20	34
25	0.64	0.64	0.64	0.63	0.43	0.35	0.42	114	20	114
26	0.73	0.52	0.52	0.72	0.33	0.17	0.32	126	52	126
27	0.66	0.67	0.67	0.65	0.265	0.22	0.261	140	52	140
28	1.13	1.13	1.13	1.12	1.26	1.24	1.29	38	36	36
29	0.05	0.05	0.05	0.05	0.0197	0.0109	0.0193	224	332	224
30	2.02	2.02	2.02	1.98	3.43	3.53	3.18	44	74	74